Changes to pedagogy

The emphasis on teaching strategies to the students resulted in changes to the ways in which the teachers taught mathematics. The teachers rethought the organisation of their mathematics sessions to facilitate a greater emphasis on questioning and the explanation of mathematical thinking and changes in the use of equipment. Changes to instructional grouping were informed by data gathered from the individual diagnostic interviews which will be discussed in a later section.

Providing opportunities for children's explanations

Many of the teachers reported taking a more active role and spending longer with a group to ensure more opportunities for teacher and student questioning and explanations. Barbara describes this as spending more time "just talking" to the students:

> Previously, I wouldn't have spent so much time just talking. There was a lot of me teaching and them giving the answers back or telling me exactly what they did to get the answer, but now there is so much talking with the children, not at the children. I think that is a good thing, but I am finding I can easily sit with one of my groups for a good half an hour and then I've only got twenty minutes left. I am wondering,

> well, is that a good use of time – I think it is, but it's that mind shift again. In the past, it might have been only fifteen minutes maximum.
>
> Barbara at Numerate School, First Interview

Barbara's comments reflect the changes that many of the teachers made to the ways in which they worked with children in an instructional group. The essence of these changes is the role taken by the teacher and the corresponding role open to the children. When a teacher takes the role of questioner or instructor, it often leads to children falling into a response mode, as in the Initiation Response Evaluation sequence identified by Cazden (1988), amongst others. When a teacher moves from this role to one of "talking with the children", then children are likely to have more opportunity to explain their mathematical thinking.

Anita commented that, in trying to emphasise the strategies, she spent more time working with different groups:

> I really focus them on the strategies in thinking about numbers and how quickly they can do things; that is important. … I make time to work with groups and talk to them about our number strategies, getting them to think in their head more than I used to.
>
> Anita at Ratio School, Second Interview

The group work provides an opportunity for students to talk about the different strategies they have used to solve a number problem in their heads, with an emphasis on efficiency.

Deirdre, a teacher trained in Reading Recovery, thought about her questioning in reading and saw this as fitting with the changed emphasis in mathematics on developing students' strategies.

> And the strategies. I mean, you can teach children how to use strategies in reading by the questions that you use. I've often wondered whether there are specific questions that you can use in maths that actually make the child think more and use specific strategies. … I suppose it's putting the onus back onto the child to explain their learning. Like when they are doing it in reading and you say to them "Does that look right, does it sound right, how would you expect it to start?" With all the strategies that you use in reading, it's the same sort of thing in maths – "Would that fit there? What else could fit there? Why did you come up with that? Can you explain it? How did you go about solving it?" All the questions that make the child really think about what they have been doing. … Reading and maths are very similar.
>
> Deirdre at Multiple School, First Interview

Deirdre makes a useful connection between her teaching strategies for reading and the possibilities of similar questioning to promote children's mathematical thinking through a focus on explanation.

Rachel commented on how emphasising student explanation had impacted on her practice of getting students to reflect on their mathematics learning for that day:

> I probably spend more time asking the kids how they got their answer, you know, what they were thinking of when they were doing it, and I think that has probably been my major change. … getting the kids to actually reflect on what they are doing and, instead of me telling them … this is what we've covered today, getting them to reflect [on] how they found that particular concept. … it's quite hard for them to

> actually explain themselves … how they got their answer – "Oh, I just did it, I just got it" – "But how did you get it?" And it's making them … stop and think – "Oh well, what did I do? What steps did I take?" So I think that has probably been a major area in the last little while. It's changing attitude … It's getting kids and ourselves to be more reflective, I think.
>
> … Well, we've always asked kids to reflect. We've often got the kids at the end of the session to reflect on how they felt they went, but I think [we are] getting more to how they work things out, I think maybe we are just pinpointing it, we are a little bit tighter on what we are reflecting on. … "Well, I found maths good today" – well, that tells you nothing, doesn't it. "What did you actually [do]?" I think it's pinpointing it more. … And when you're sharing with kids … it's all good experience, [for example] some kids say "Oh, you know, I worked it out – 2, 4, 6, 8, 10 and so on" and another kid says "Oh, I worked it out this, this, this way" … it's imparting their knowledge to the other kids as well … it's giving other kids who perhaps haven't got their thinking at that level, something to hang on to.
>
> Rachel at Even School, First Interview

Rachel's description of developing students' explanations from the initial response of "oh, I just did it, I just got it" or "I worked it out in my head" to being able to explain the steps they took to solving the problem signals an orientation towards the processes involved in solving problems. She describes this as "changing attitude". The focus on children's explanation carries over to the common practice of asking children to reflect on what they have learned in mathematics for that day. She points out that global comment such as "well, I found maths good today" tells the teacher and children (and parents) very little compared to recording the strategies they used that day to solve a problem. She also points out that an added benefit of sharing explanations is to introduce other children to possible strategies that they might use in future to solve problems. Rachel comments again in her second interview on the children's ability to explain their mathematical thinking to others as a result of the teachers' focus on strategic thinking.

Use of equipment

The teachers mentioned a greater range and increased use of equipment as a result of their involvement in the Numeracy Exploratory Study. This equipment included the number line, bead frames, popsicle sticks, ten-dot cards, and expanded numeral cards. What was striking about the teachers' references to this equipment was that, without further prompting from the interviewer, they invariably gave examples of an equipment item and suggested a mathematical purpose for its use. The number line was the most frequently mentioned and fully discussed by all the teachers, particularly in terms of its usefulness in developing students' strategic thinking. This section will discuss the various ways in which the number line was used.

Roger described the different sorts of number lines in terms of supporting the students' move from concrete to abstract:

> The number line gives them something a bit concrete and it moves from a filled-in number line with all the numbers on and so … it goes right through to 100, then it moves to the next stage, which is a number line with divisions but no numbers, and

> then it leads to the kids drawing their own number line, and I think it's just moving from the concrete to the abstract, basically.
>
> Roger at Unit School, Second Interview

Roger goes on to explain how he used the number line to teach subtraction:

> I've tried to get the students to realise that what they are looking for is a difference between the two numbers, so they know what they are doing. 54 take away 29: in between here there is a number and that's what we are trying to find out. … Well, say it's 54 and they've got to take off 29, well, they put their little 29 on the number line here and 54 there. What they do is they go to the nearest ten number, so they do a little bunny-hop jump and they write the 1 there. Depending on what stage they are at, they scoot along to 50 and they might go 10, 10 or they might, if they are further along, 20, then they bunny hop along to 4, 54, and then they just add up these top numbers.
>
> … they are getting to the stage, some of them, where they can do it without a number line and they can start when they go 29, one's 30, 20, 54 and add it up like that in their head, so I guess they're at a stage where we are at. … I've been quite surprised how well they can do it. … I would start them … with the number line and do the explaining of how we are looking for the difference and we need to go from that number to that number. … Well, I told the kids it was backwards addition. I think it's addition going backwards, in reverse, basically.
>
> Roger at Unit School, Second Interview

The number line is particularly useful for illustrating the idea of subtraction as a difference between two numbers, as opposed to the way the vertical algorithm shows subtraction as taking away a quantity from a number. Roger suggests the strategies that children might use to find this difference by working through the decade numbers. He is also able to make the links between addition and subtraction.

Deirdre explained the specific questions that she would ask which might lead to the children thinking more and using particular strategies:

> … being able to do things mentally like 51 minus 36 and turn it around the other way, take 36, how many to 40 and then jump to 51, and that's what we've been working with in the classroom, using number lines. Well, there are specific questions you could say to them, if [they] are going to jump to the next round number, the next easy step: "How many would you need?" You can ask them that kind of question and that helps.
>
> I do examples like: "What number would you expect to see between 17 and 19?" I know that's really easy, but that's the kind of questioning I would use now, whereas, in the past I would have said: "What number comes after 17?" or "What comes before 19?" I would never have thought of using "in between", "comes between", or: "If you are taking 3 away from 12, what number would you start at? Put your finger on the number that you are going to start at, now count back, how many would you need to count back? Where are you going to end up? And if you are going from 9 up to 12, where would you start? What number would you start at?"
>
> Deirdre at Multiple School, Second Interview

Using a similar example to Roger, Deirdre is able to identify the questions that might help children to solve a problem mentally, for example, 51 minus 36. As with Roger, Deirdre also is able to turn the problem around to show the links between addition and subtraction. In her questioning, she encourages the students to look for "friendly numbers" or what she terms "the next round number, the next easy step", such as using multiples of ten. She gives the example of "36, how many to 40?".

The number line is useful in helping children to visualise or imagine the numbers in their heads, overcoming, as Lynley terms it, "the hurdle of visualisation":

> … some of the kids can't visualise, they are still working with a lot of concrete stuff, they can't visualise the different strategies in different ways of doing things at all, so they are struggling with the hurdle of visualisation.
>
> Lynley at Unit School, Second Interview

Annette described in some detail how she teaches children to image numbers, as part of the process of moving from relying on concrete referents to developing the ability to think abstractly:

> … 91 minus 36. So we started with a number line that actually had numbers on it … they would go up to 40 and go up to the nearest 10. When they first started, they had to do jumps of 10, but they quickly overcame that. So okay, they'd see that. Then you teach them to image it in their head. "So, okay now, close your eyes and think in your head, where would you jump to?" But then there are other strategies they can use too. … they would just work that out in their heads. They understand what's actually happening. … It's like teaching a trick almost sometimes, because you get to that stage, and … you are teaching this to children who are not really brilliant. Following this process through, it's not until your children are able to do this, image this, and then you … say to them "And sometimes we need to use an algorithm, and this is what you do".
>
> Annette at Multiple School, Second Interview

Annette suggests that it is through developing an ability to image numbers in their heads that children develop their understanding of number. As she puts it "it's not until your children are able to do this, image this" that the written algorithm should be introduced.

The diagnostic interview as a trigger for change

In the first interview, many of the teachers spoke of the importance of the diagnostic interview in changing their practice. In some cases, this may have been because the teachers' judgements of students' knowledge were challenged. It appeared that the interview was a key to shifting the teachers towards an emphasis on children's strategic thinking. As a consequence of getting data on student achievement, the teachers commented that they were more frequently asking for, and listening to, the students' explanations. They also reported feeling more confident and having a clearer focus on their teaching goals. The teachers felt they had a better idea about the students' understanding of number, with many speaking of gaining more knowledge about students' understanding of number concepts as a result of the diagnostic interview. In some cases, this information did not match up with their perceptions and assumptions about their students' abilities. The comments of Marcy, and Rachel typify the impact of the results on many of the teachers:

> I was quite motivated by seeing what process they are actually doing [and] learning themselves. … I was just so keen to see if my expectations of them would be the same as what would be [seen] and how they would act with these tests.
>
> Marcy at Ratio School, First Interview

> Oh, I think the test very clearly enables a teacher to see where that child is, whether they are counting on, or whether they have got all the strategies like combining, you know, multiplication strategies and things, instead of repetitive addition. I've never done anything that I can recall that's actually pinpointed the strategies as clearly as this.
>
> Rachel at Even School, First Interview

Many of the teachers were interested in having an opportunity to check out their judgements of children's abilities to engage in strategic thinking and, as Rachel points out, many of the traditional forms of pencil and paper tests do not provide such an opportunity. Barbara described the impact of the diagnostic interview upon her:

> In fact, some of the results really blew me away, it was the processes, what they knew … I had children who I had expected to do better, but they didn't, and I had some children who I had expected to be at a certain level and [they] ended up quite high up in terms of their cognitive processes, but because they didn't give me the answer, I just assumed, oh no, you're this level. … It got rid of all my pre-conceived ideas about where they were. Yeah, there are things that I have changed that I would never have done before if it wasn't for the testing and finding out exactly where my children were at. Yeah, things have changed.
>
> Barbara at Numerate School, First Interview

By the processes, Barbara possibly means the ability of the children to communicate mathematical ideas. The diagnostic interview emphasises mental processes rather than the pencil and paper approach of solving algorithms.

Many of the teachers were critical of their previous assessment practices of a pre- and post-test in terms of the information these gave them for grouping students for instruction and planning the next learning steps. Cate and Lynley both contrast the interview with the pre-and post-test that they had used previously:

> Rather than just a pre-test and: "they can do this and they can't do this", [it's] "how are they getting to that point, what are they thinking to get there?"
>
> Cate at Even School, First Interview

> … [with] pre-tests or anything like that … even … if you do get a lot out of it, you don't actually get the verbal steps from them, so they might be able to solve the problem but whether they actually understand the steps in that actual mathematical concept is totally different.
>
> Lynley at Unit School, First Interview

The emphasis in the diagnostic interview is on the students' thought processes in arriving at a solution. Lynley suggests that such information is more likely to provide an opportunity to gauge student understanding than traditional written forms of assessment. It is the importance of such information in planning the next learning step that is crucial to the planning process.

Both Barbara and Erica commented on the usefulness of the information from the diagnostic interview:

> If you don't know where they are at, well, you don't know what you are doing.
>
> Barbara at Numerate School, Second Interview

> I highlighted all the gaps and holes, and so I knew what I needed to teach them and what each child needed to work on, so it has been really helpful …
>
> Erica at Numerate School, First Interview

In terms of grouping students, Barbara realised that, by changing her assessment practices, the basis for her grouping had changed from knowledge to the processes that students were using. Her comments in the first and second interviews reflect this shift.

> I used to think levels, levels, levels, but the levels have gone out the window … I remember the facilitator asking "How do you group your children for number?" "Oh, I do a readiness test." "Oh yeah, what does the readiness test tell you?" "Oh, what level they are at." But when we sat down and analysed all the test results, they didn't fit into those levels.
>
> Barbara at Numerate School, First Interview

> I can group them now, but I know it's based more on strategy than knowledge.
>
> Barbara at Numerate School, Second Interview

The shift from an emphasis on knowledge to an emphasis on strategies is critical. As discussed in Chapter 3 (page 17), this is perhaps a false dichotomy as children's growth in knowledge is enhanced by the development of their strategic thinking and vice versa. Nonetheless, making such a distinction is useful in discussing the essence of the shift in emphasis from classroom mathematics programmes based on knowledge to programmes that incorporate both knowledge and strategies. In practical terms, such a distinction enables teachers to rethink the basis of their instructional grouping of children from one informed only by the extent of children's knowledge of mathematics to one that is also informed by a consideration of the children's abilities to think strategically when solving mathematical problems. The goal is for children to develop increasingly sophisticated and efficient approaches to solving number problems.

Rachel reflects back on her previous assessment practice of a pre- and post-test:

> In the past, I've probably done a pre-test and worked out these kids know this and so they are in one group, and these kids, and so on, but we are probably now looking – especially with number – we are looking at the kids who are still counting with their fingers, [who] aren't counting on, and we're looking at grouping them according to their strategy rather than, perhaps before, we might have just grouped them according to what their knowledge was. … Yeah, I think it's a huge shift, and it's hard to work out the grouping in your class, I think. We are still working on it, but that's where that testing enables us to slot those kids into groups much more easily. … I think the testing has given me a lot more understanding of what I should be teaching … specifically – specifics.
>
> Rachel at Even School, First Interview

An added benefit of the interview was that it also provided teachers with information and a language to describe students' progress both to other teachers and to parents. Erica and Rachel explain:

> … now you can actually label the progress children are making and know where to teach them and where to move on from that … I know so much that I can practically pinpoint where they are up to and where to take them on.
>
> Erica at Numerate School, First Interview

> Yeah, I think [the diagnostic interview is] a very useful tool. And then we can be more specific, and we can be more specific with our parents, and say, "Well look, Johnny needs practice in trying to develop mentally, he's still using his fingers, so maybe we can think of other ways instead". … I will show them the forms because I think it is important that parents can see what the strategies are too, because a lot of parents can be quite useful in helping kids: "oh let's count in fives" … so they can see the strategies that we are aiming for.
>
> Rachel at Even School, First Interview

Having a language to describe the specific stages of children's progress, rather than the global statements often used, enables teachers to pinpoint children's progress more accurately in reporting to the children themselves, to other teachers, and to parents. As Rachel comments, this means that "parents can see what the strategies are too, because a lot of parents can be quite useful in helping kids: 'oh let's count in fives' … so they can see the strategies that we are aiming for". Reporting then becomes another form of parent education.